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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2016

FIRST YEAR [BATCH 2016-19]

Date : 19/12/2016 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper : I** Full Marks : 75

[Use a separate Answer Book for each Group]

Group - A

(*Answer any five questions*) [5×5]

1. a) If n be a positive integer, prove that $(1+i)^n+(1-i)^n=2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$. [2]

   b) Find Log z and log z, where $z=1+i\tan\theta$, $\frac{\pi}{2}<\theta<\pi$. [3]

2. Show that the product of all the four values of $(1+i\sqrt{3})^{3/4}$ is 8. [5]

3. Solve the equation : $x^4-10x^3+29x^2-22x+4=0$, if one of its root is $(2+\sqrt{3})$. [5]

4. Solve the equation by Cardan's method : $x^3-18x+35=0$. [5]

5. Prove that $\begin{vmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{vmatrix}=abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. [5]

6. Solve by Cramer's rule : $x+2y+3z=6$, $2x+4y+z=7$ and $3x+2y+9z=14$. [5]

7. Find the rank of the matrix : $A=\begin{pmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 5\\ -1 & -2 & 6 & -7\end{pmatrix}$. [5]

8. Examine the consistency of the following system of equations and solve, if possible,
   $x+y+z=1$, $2x+y+2z=2$ and $3x+2y+3z=5$. [5]

Group - B

(*Answer any five questions*) [5×5]

9. If $\mathbb{Z}$ be the set of all integers and A, B, C are subsets of $\mathbb{Z}$ given by
   $A=\{x\in\mathbb{Z}\mid 1\le x\le 10\}$, $B=\{x\in\mathbb{Z}\mid 6\le x\le 15\}$, $C=\{x\in\mathbb{Z}\mid 3\le x\le 12\}$,
   verify that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$. [5]

10. a) Let $f:\mathbb{R}\to\mathbb{R}$ be defined by f(x) = 2x + 3. Show that f is a bijective mapping, where $\mathbb{R}$ is the set of all reals. [3]

    b) Is the mapping $f:\mathbb{Z}\to\mathbb{Q}$ (set of all rationals) defined by f(x) = 2x – 1 onto? [2]

11. Prove that under usual matrix addition and multiplication
    $M=\left\{\begin{pmatrix}a & b\\ 2b & a\end{pmatrix}: a,b\in\mathbb{Q}\right\}$ forms a field. [5]

12. Prove that the set $A=\{(1,1,2),(2,1,1),(1,2,1)\}$ is a basis of $\mathbb{R}^3$. [5]

13. Verify Cayley-Hamilton theorem for $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$. Hence find its inverse. [4+1]

14. Find the rank, index and signature of the following quadratic form :
$2x^2 + y^2 - 3z^2 - 8yz - 4zx + 12xy$. [5]

15. a) If λ is an eigen value of a non-singular matrix A then prove that λ^m is an eigen value of A^m, where m is a positive integer. [2]

b) Show that the vectors (2, 4, 0), (0, 1, 0) and (2, 6, 2) are linearly independent in the real vector space $\mathbb{R}^3$. [3]

16. Determine the eigen values of A, where
$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Find eigen vectors corresponding to the eigen value 4. [3+2]

Group - C

(*Answer <u>any five</u> questions*) [5×5]

17. a) Find $\lim_{x \to 0}(1+2x)^{\frac{x+3}{x}}$. [2]

b) Find the value of *a* if $\lim_{x \to 1} f(x)$ exists, where $f(x) = 1 + \frac{2x}{a}$, if $0 \le x < 1$
$= ax$, if $1 \le x < 2$. [3]

18. a) Show that $f(x) = \frac{|x|}{x}$ for $x \neq 0$
$= 1$ for $x = 0$
is discontinuous at x = 0. [2]

b) If f(x) be an even function of x and $f'(0)$ exists, show that $f'(0) = 0$. [3]

19. Show that $\frac{x}{1+x} < \log(1+x) < x$, if x > 0. [5]

20. State Leibnitz's theorem on the derivative of the product of two functions of x and use it to prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0$, for the function $y = \sin(m \sin^{-1} x)$. [1+4]

21. If u = $F(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. [5]

22. Find the pedal equation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. [5]

23. Show that the radius of curvature of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at the point $\theta = 0$ is 4*a* and at the point θ, $\rho = 4a\cos\frac{\theta}{2}$. [5]

24. If $f(x, y) = \frac{x^3 + y^3}{x - y}$, if $x \neq y$
$= 0$, if x = y,
show that f(x, y) is not continuous at (0, 0). [5]

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